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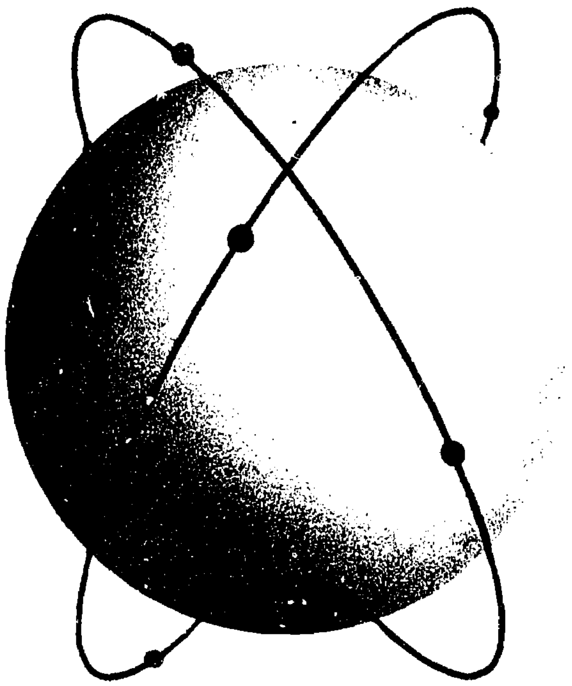
ABSTRACT

As the ninth lesson of the Articulated Multimedia Physics Course, instructional materials relating to universal gravitation are presented in this study guide. The subject is concerned with the quantitative meaning of the law of universal gravitation and its applications in astronomy. The content is arranged in scrambled form, and the use of matrix transparencies is required for students to control their learning activities. Students are asked to use magnetic tape playback, instructional tapes, and single concept films at the appropriate place in conjunction with the worksheet. Included are a homework problem set, a study guide slipsheet, and illustrations for explanation purposes. Related documents are SE 015 963 through SE 015 977. (CC)

ARTICULATED MULTIMEDIA PHYSICS



LESSON 9

NEW YORK INSTITUTE OF TECHNOLOGY
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ARTICULATED MULTIMEDIA PHYSICS

Lesson Number 9

UNIVERSAL GRAVITATION

IMPORTANT: Your attention is again called to the fact that this is not an ordinary book. It's pages are scrambled in such a way that it cannot be read or studied by turning the pages in the ordinary sequence. To serve properly as the guiding element in the Articulated Multimedia Physics Course, this Study Guide must be used in conjunction with a Program Control equipped with the appropriate matrix transparency for this Lesson. In addition, every Lesson requires the availability of a magnetic tape playback and the appropriate cartridge of instructional tape to be used, as signaled by the Study Guide, in conjunction with the Worksheets that appear in the blue appendix section at the end of the book. Many of the lesson Study Guides also call for viewing a single concept film at an indicated place in the work. These films are individually viewed by the student using a special projector and screen; arrangements are made and instructions are given for synchronizing the tape playback and the film in each case.

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New York Institute of Technology
Articulated Multimedia Physics

LESSON 9

STUDY GUIDE SLIP SHEET

STUDY GUIDE TEXT:

<u>Page 24</u>: Top of page, equation on right side, change $\frac{1}{k^2}$ to $\frac{1}{r^2}$

<u>Page 57</u>: Cross out "Before continuing", then add to the sentence so that it now reads: "Please turn to page 123 in the blue appendix <u>after</u> you have read the descrip- of the Cavendish apparatus given below." You will find it helpful to learn something about the construction of the equipment that you're going to see on the film called for in the blue appendix.

STUDY GUIDE DIAGRAMS: No changes.

WORKSHEETS: The Worksheet for Tape Segment 4 has been erroneously printed on the back of the pink Homework sheet. Please look for this Worksheet in that place. It now carries the label "p. 124".

HOMEWORK PROBLEMS:

<u>Problem 7</u>: Signify that this problem is to be omitted by crossing it out. You will find this problem given in the Homework section of Lesson 10. Change number 8 to number 7.

<u>Problem 3</u>: Second line, toward right, change "mody" to "body".

Upper left corner heading: Change to AMP LESSON 9.

During the two plague years of 1665 and 1666, Isaac Newton, who had just completed his first degree at Trinity College, Cambridge University in England, left the campus to study in isolation at his home in Woolsthorpe. During these years, he developed a clear concept of the first two laws of motion discussed in our previous lesson, and succeeded in deriving the equation for the force exerted on a rotating object. It is highly probable that the famous incident of the falling apple also occurred during the same two-year interval.

If you have been thinking that the falling-apple story is in the same category of imaginative tales as the one about George Washington and the cherry tree, you're probably wrong. William Stukely, a friend of Newton's, published in 1752 the scientist's biography which records the following incident: On one occasion when Stukely was having tea with Newton in a garden under some apple trees, Newton casually informed him that "he was just in the same situation, as when formerly, the notion of gravitation came to his mind. It was occasion'd by the fall of an apple as he sat in a contemplative mood. . ."

We must not interpret this statement to mean that progress in science depends upon such accidental and trivial happenings as the fall of an apple from a tree. Newton was in a "contemplative mood"; the falling apple merely served as a trigger which turned Newton's thoughts to gravitation.

Please go on to page 2.

In this lesson we shall study the quantitative meaning of the Law of Universal Gravitation. We expect that you will understand this material thoroughly because we shall present it through the medium of your grasp of proportionalities; your understanding will be strengthened by working out some interesting problems dealing with weight changes and similar effects.

Perhaps, as you study this lesson, you will join us in the feeling of awe we experience each time we meditate on the tremendous intellect of Sir Isaac Newton. Today in universities and laboratories throughout the world, physicists and engineers draw up plans for new space vehicles, new types of satellites, and safe interplanetary spaceship--all basically dependent on the laws of physics first expounded by a man who was a student like you about 300 years ago.

Allow your imagination free rein. If you have a normal amount of curiosity and a desire to understand your universe, you should derive considerable enjoyment from following us as we carefully retrace the thinking that led to this great generalization of physics.

Please go on to page 121 in the blue appendix.

You probably began to hear about gravity and gravitation when you were in the elementary grades. At first you accepted the force of attraction between two bodies of matter, like the apple and the Earth, as quite natural. You didn't have to explain it; you just knew it was there. Infants seem to be born with an instinctive fear of loss of support, a fear of falling, so this is something that doesn't have to be learned.

Yet as you grew older and more mature, you probably began to ask why bodies should attract each other, why there is a force of gravitation wherever matter exists. It's regrettable but true that we cannot yet answer these questions. We just don't know the source of the gravitational force. There are hypotheses, of course, and each passing year sees an improved understanding of this phenomenon, but we have a long way to go before any of these hypotheses can be accepted. So we must be content with being able to describe gravity with great rigor, with being able to put gravity into mathematical terms, and with being able to use our knowledge to practical advantage.

We know that the masses of two nearby bodies influence the force of gravitation between them. To be sure you remember just what mass means, look over the following statements and then choose the one you think is most correct:

(1)

A Mass is the same thing as weight.

B The larger the space occupied by a body, the greater its mass.

C A large quantity of matter has a correspondingly large mass.

YOUR ANSWER --- C

You are correct. More than 100 years after Newton published the law of gravitation in his "Principia," Henry Cavendish, another British scientist, performed a laboratory experiment in which he showed that a situation such as that depicted in Figure 1(Z) on page 99 would involve a larger gravitational force than either that of X or Y.

Let's imagine that Cavendish used our figures (Figure 1 on page 99) for the masses, keeping r the same in each case. If he did, he might have gotten results like those shown in Figure 2.

	CASE X	CASE Y	CASE Z
m_1	1 kg	1 kg	10 kg
m_2	10 kg	1 kg	10 kg
F_g	10 UNITS	1 UNIT	100 UNITS

F_g = FORCE OF GRAVITATION BETWEEN m_1 AND m_2 MEASURED IN SOME ARBITRARY UNIT (NOT NEWTONS)

Figure 2

The masses exert mutual gravitational attraction on each other; this is F_g, measured in some arbitrary unit (not newtons). Study the relationships in each vertical column; then compare them with each other. From these results it appears that, with r constant, the force of gravitation between bodies is directly proportional to which of the following?

(3)

A The product of their masses.

B The sum of their masses.

C The quotient of their masses.

YOUR ANSWER --- C

You are correct. Originally we had:

$$F_g = km_1m_2 = 16 \text{ units}$$

When each mass is doubled, we then have:

$$F_g = k(2m_1)(2m_2) = 4km_1m_2$$

Hence, the new force is four times larger than the original force or 4 x 16 units = 64 units.

Tiros II, an artificial satellite, is kept in orbit around the Earth by virtue of the gravitational attraction between the two bodies. Suppose Tiros II had been built with 10 times its present mass and placed in the same orbit. What would then be the gravitational attraction acting between the satellite and Earth?

(5)

A 100 times as large as it is at present.

B 10 times as large as it is at present.

C The same as it is at present.

YOUR ANSWER --- B

You are correct. Even if you knew nothing at all about gravitation, this would be a sensible guess. Gravitation, like magnetism, works over a distance. The force of gravitation, like the force of magnetism, decreases as the distance of separation is increased.

We can't take time now to explain how he did it, but Newton was able to show that the force of gravitation decreases as the square of the distance between centers of the masses. Cavendish's measurements also showed that the force of gravitation is inversely proportional to the square of the separation distance. Both of these statements convey the same information.

We want this relationship stated mathematically. Which one of the following is the proper way to write it?

(7)

A $F_g = kr^2$

B $F_g = k/r$

C $F_g = k^2/r$

D $F_g = k/r^2$

YOUR ANSWER --- B

You forgot to square the "r" factor.

Remember, when the distance is doubled, the "2" multiplier must go inside the parenthesis for r. Then, when you square the denominator, it becomes $4r^2$, not $2r^2$.

Please return to page 97. Select the answer which takes into account a numerator multiplied by 3 x 4 and a denominator multipled by 4.

YOUR ANSWER --- A

You are correct. Although Callisto has a slightly greater mass than Io, it is so much farther away from Jupiter that the distance factor more than makes up for the greater mass.

Refer to Figure 3 on page 65.

Working to two significant figures, determine how many times as great the gravitational force acting on Io is compared to that acting on Callisto, with Jupiter taken as the second gravitating mass in each case. Use m_J for the mass of Jupiter, 1 (unit) for the mass of Io, and 1.3 (units) for the mass of Callisto. Since we are setting up a ratio of factors having identical units, the units may be omitted altogether. Work out the solution fully; then select one of the answers below.

The gravitational force acting on Io compared to that acting on Callisto is which of the following?

(10)

A Between 2 and 10 times as great.

B Between 10 and 100 times as great.

C Between 100 and 1,000 times as great.

D Not within any range given above.

YOUR ANSWER --- A

You don't mean it!

The pound is an English unit and does not fit in[illegible]e MKS system at all!

Please return to page 55. Think before you make your next choice.

YOUR ANSWER --- A

This is a discouraging answer!

It suggests that we are going to have to invent a new unit to measure the force of gravitation. Doesn't this also suggest that such a problem is likely to occur again and again, every time we derive a new equation for different kinds of forces?

Can you imagine the mess we would find ourselves in if we allowed this to happen?

There simply must be a way out of the difficulty. Try to see what that way must be.

Please return to page 106. The answer is there.

YOUR ANSWER ---- C

You are correct. The steps in the simplification follow:

We start with: $$k = \frac{\frac{kg\text{-}m}{sec^2} \times m^2}{kg^2}$$

We next multiply $\underline{m}$ x $\underline{m^2}$, obtaining m^3. Also, we cancel the $\underline{kg}$ in the numerator against one of the $\underline{kg's}$ in the denominator giving us:

$$k = \frac{\frac{m^3}{sec^2}}{kg} \text{ or } \frac{m^3}{sec^2} \times \frac{1}{kg} \text{ by the reciprocal rule.}$$

So finally we have:

$$\frac{m^3}{kg\text{-}sec^2}$$

The unit is read as: "meters cubed per kilogram second squared."

A unit like this has little intrinsic value. It doesn't call pictures up in your mind like the simpler ones. For instance, when you say that the speed of a car is 60 mi/hr, you have a definite visualization of a mile of distance being traversed in a minute of time. But you can't apply the meter cubed per kilogram second squared in the same way; it lacks the clear-cut conceptual meaning of the basic units for distance, area, time, density, and so on.

Once this is recognized, you don't need to concern yourself with it any longer. Realize that it is an artificial unit created with a specific purpose in mind: to allow you to measure mass in kilograms and distance in meters and then have the force of gravitation come out in newtons.

Please go on to page 12.

Just as a check, let's substitute the units back into the original gravitation equation to ascertain that F_g does come out in newtons.

$$F_g = k\frac{m_1 m_2}{r^2} = \frac{m^3}{kg\text{-}sec^2} \times \frac{kg \times kg}{m^2}$$

Then, clearly, the $\underline{m^2}$ in the denominator takes out an $\underline{m^2}$ from the $\underline{m^3}$, leaving $\underline{m}$ in the numerator; one kg takes out the kg in the denominator; this leaves finally kg-m/sec^2 which, of course, is the newton.

So, the constant in Newton's Second Law is a pure number, but the constant in the Law of Universal Gravitation has units. There is still another difference, however. In the Second Law, the newton was defined by making $\underline{k}$ equal unity; that is, if $\underline{m}$ is in kilograms and $\underline{a}$ is in m/sec^2, then the force in F = kma is in newtons. Hence, not only is $\underline{k}$ dimensionless (a pure number), but it is equal to 1.

Please go on to page 13.

Now let us assume for the moment that k in the Law of Universal Gravitation is 1 $m^3/kg\text{-}sec^2$. We repeat, this is an assumption only; it must be tested. Then suppose you had two 1 kg masses separated by exactly one meter between their centers.

How large would be the gravitational force between them?

(14)

A 0.1 newton.

B 10^{-2} kilograms.

C 1 kilogram.

D None of the above is correct.

YOUR ANSWER --- A

Review the meanings of the symbols in:

$$F_g = k\frac{m_1 m_2}{r^2}$$

Cavendish knew the distance between the centers of the masses; hence he knew r. If a quantity is known, it cannot be an unknown, can it?

Please return to page 57. You can do better tban this.

YOUR ANSWER --- B

The numerical portion of this answer is wrong.

You made an error in handling your powers of 10. It appears that, in moving the 10^1 kg^2 in the denominator up to the numerator, you forgot to change the sign of the exponent.

Correct the error; then please return to page 76 and make another selection.

YOUR ANSWER --- A

There is nothing wrong with this statement of the Second Law; it is a perfectly good general expression of the relationship between acceleration, force, and mass.

The trouble is, you were supposed to select the best statement of the Second Law as applied specifically to bodies in free fall. For this special case of the Second Law, we symbolize acceleration due to the Earth's gravitational pull by g rather than a. Furthermore, the force in this case is certainly a very specific one, namely the force of gravitation.

There is a much better expression in the list for bodies in free fall.

So please return to page 32 and find the best expression.

YOUR ANSWER --- C

You can't do that.

The symbol *g* means gravitational acceleration which, at the surface of the Earth, is 9.80 m/sec^2. The symbol G means the constant of universal gravitation which is 0.667×10^{-10} $m^3/kg\text{-}sec^2$.

Since these are two entirely different quantities, they cannot be eliminated by cancellation.

So please return to page 11. The necessary operation is really quite simple.

YOUR ANSWER --- C

Fine! You are correct. Since the mass of the object has dropped out of the equation (when the m_o's were canceled), we know that g can no longer be affected by this quantity. Thus we have shown why it is that all bodies (discounting air resistance) have the same acceleration, i.e., 9.80 m/sec^2 or 32 ft/sec^2, near the surface of the Earth.

NOTEBOOK ENTRY
Lesson 9

(Item 1)

(e) All bodies, regardless of their mass, have the same acceleration near the surface of the Earth (9.80 m/sec^2 or 32 ft/sec^2). This follows from:

$$g = \frac{F_g}{m_o} \quad \text{where } F_g = \text{force exerted on body by Earth and } m_o = \text{mass of body}$$

But since $$F_g = G\frac{m_o m_e}{r^2}$$

THEN $$g = \frac{G\frac{m_o m_e}{r^2}}{m_o} = G\frac{m_e}{r^2}$$

Since m_o does not appear in the final expression for g, then the mass of the object cannot affect gravitational acceleration.

Now for a little fun. How would you like to calculate the mass of our planet, the Earth? To help you go through the operations, we are going to do the tedious arithmetic for you. Of course, if you work on a slide rule, we'd be happy to have you check each calculation.

Before continuing, however, please turn to page 124 in the blue appendix.

Ready? Let's start with the equation we previously obtained as an expression of Newton's Second Law for bodies in free fall.

$$g = G\frac{m_e}{r^2}$$

Now let's see. We know the values of g, G, and r. That means that the sole unknown is the mass of the Earth, m_e.

So let's solve the literal equation for m_e. Go ahead. Do it.

When you answer is written on paper, turn to page 113.

YOUR ANSWER --- C

You are absolutely correct.

Completing the tedious part of the arithmetic, let's obtain the result of this operation:

$$\frac{9.8 \times 4.1}{6.7} = 6.0$$

So, what is the final answer for the mass of the Earth in kilograms?

Write your answer on scrap paper; then turn to page 114, please.

YOUR ANSWER --- B

You are correct. So, we'll write down the values of the knowns in the above statement and then solve. (Incidentally, the mass and radius of Mars have been determined with good precision by both astronomical and physical methods.) We'll work, as before, to two significant figures.

Given:
m_p = mass of person = 100 kg
m_m = mass of Mars = 6.4×10^{23} kg
r_m = radius of Mars = 3.4×10^6 m
G = gravitational constant = 6.7×10^{-11} m^3/kg-sec^2

The substitutions:

$$w = 6.7 \times 10^{-11} \times \frac{100 \times 6.4 \times 10^{23}}{(3.4 \times 10^6)^2}$$

Simplifying and collecting similar terms:

$$w = \frac{6.7 \times 6.4}{(3.4)^2} \times 10^{??}$$

You tell us. What is the value of the exponential term?

(23)

A 10^0

B 10^2

C 10^8

YOUR ANSWER --- D

You are correct. If solved for <u>k</u>, this expression yields:

$$k = F_g r^2$$

It's obvious that F_g and r^2 form a <u>constant product.</u> This, as you know, is a requirement for an inverse proportion.

NOTEBOOK ENTRY
Lesson 9

(Item 1)

(b) If the masses of two bodies are held constant, the gravitational attraction between them is inversely proportional to the square of the distance separating their centers.

$$F_g = \frac{k}{r^2}$$

Don't put your notebook away quite yet. Read over the two parts of the Law of Universal Gravitation. Newton combined these two parts into a single statement which, in our modern terminology, reads as follows:

NOTEBOOK ENTRY
Lesson 9

(Item 1)

(c) Thus, every body in the universe attracts every other body with a force that is directly proportional to the product of their masses and inversely proportional to the distance between their centers.

$$F_g =$$

Do you think you can finish the equation without the help of choices? Try it. Write it on scrap paper. If you have forgotten how to combine two proportions into a single one, review notebook entries Lesson 8, 2(a) through 2(d).

Please turn to page 95 for the correct answer.

YOUR ANSWER --- A

You are correct. We hope you chose this equation on your first try. Good work. The two equations we have now are:

At Sea Level:	At the New Altitude:
$w = G\frac{m_r m_e}{(4,000)^2}$	$\frac{w}{2} = G\frac{m_r m_e}{r^2}$

According to the terms of the problem, which quantities on the right side of the equation remain constant for any altitude? Before choosing your answer, write both equations on your scrap paper in the form given above. They may not be presented again until much later.

(25)

A G, m_r, and r.

B m_r, m_e, and r.

C m_e, G, and m_r.

YOUR ANSWER --- A

You are correct. Good. This answer is obtained as follows:

$$\frac{w = \dfrac{k}{(4,000)^2}}{\dfrac{w}{2} = \dfrac{k}{r^2}} \quad \text{simplifies to} \quad \frac{1}{\dfrac{1}{2}} = \frac{\dfrac{1}{(4,000)^2}}{\dfrac{1}{k^2}}$$

It can be reduced to:

$$2 = \frac{r^2}{(4,000)^2}$$

Now, to find r (the distance between the center of the rocket and the center of the Earth at the new altitude) we simply need to solve this last equation for the unknown. Do it; then choose the correct answer below. (Remember that you want r; hence, when you solve for r^2, you will have to take the square root of both sides.)

(27)

A $r = \sqrt{\dfrac{2}{(4,000)^2}}$

B $r = \dfrac{1.41}{4,000}$

C $r = \dfrac{4,000}{1.41}$

D $r = 1.41 \times 4,000$

You have now completed the study portion of Lesson 9 and your Study Guide Computer Card and A V Computer Card should be properly punched in accordance with your performance in this Lesson.

You should now proceed to complete your homework reading and problem assignment. The problem solutions must be clearly written out on $8\frac{1}{2}$" x 11" ruled, white paper, and then submitted with your name, date, and identification number. Your instructor will grade your problem work in terms of an objective preselected scale on a Problem Evaluation Computer Card and add this result to your computer profile.

You are eligible for the Post Test for this Lesson only after your homework problem solutions have been submitted. You may then request the Post Test which is to be answered on a Post Test Computer Card.

Upon completion of the Post Test, you may prepare for the next Lesson by requesting the appropriate

1. study guide
2. program control matrix
3. set of computer cards for the lesson
4. audio tape

If films or other visual aids are needed for this lesson, you will be so informed when you reach the point where they are required. Requisition these aids as you reach them.

Good Luck!

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YOUR ANSWER --- B

No, it would not.

As the equation $g = G \times m_e/r^2$ indicates, g is inversely proportional to the square of the distance between the centers of gravitating bodies. Since a rocket that has gone up to 4,000 miles in altitude has doubled its distance to the center of the Earth (remember the radius of the Earth is about 4,000 miles), then the value of g would be cut down to ¼ of its value near the surface.

Please return to page 112 and select the other answer.

YOUR ANSWER --- D

You are quite correct. Here is the solution:

Starting with: $F_g = k\frac{m_1 m_2}{r^2}$

Solving for k: $k = \frac{Fr^2}{m_1 m_2}$

Substituting: $k = \frac{0.667 \times 10^{-7}\ \text{nt} \times (10 \times 10^{-2}\ \text{m})^2}{1\ \text{kg} \times 10\ \text{kg}}$

$$k = 0.667 \times 10^{-7}\ \text{nt} \times 10^{-2}\ \text{m}^2 \times 10^{-1}\ \text{kg}^2$$

$$k = 0.667 \times 10^{-10}\ \frac{\text{m}^3}{\text{kg-sec}^2}$$

Note that k is an extremely small quantity. When you stop to think for a moment, you can understand why this must be so. The Earth is a tremendously massive body (about 6×10^{24} kg!); yet it attracts a 1 kg mass with a force of only 9.80 newtons. When you compare the mass of the Earth with a 10 kg mass, it is evident that the latter must exert a much, much smaller attraction on a 1 kg mass. Hence, the force in this case is only 0.667×10^{-7} nt for a separation distance of 1/10 of a meter. The contrast becomes even stronger when you remember that the center of the Earth is at least 4,000 miles from the center of a 1 kg mass near its surface.

NOTEBOOK ENTRY
Lesson 9

(Item 1)

(d) The value of the gravitational constant k (often written as G) is $0.667 \times 10^{-10}\ \text{m}^3/\text{kg-sec}^2$.

Please go on to page 32.

Where $G = 0.667 \times 10^{-10}$ or 6.67×10^{-11} $m^3/kg\text{-}sec^2$, m_1 and m_2 are expressed in kilograms, and r in meters, the full statement of the Law of Universal Gravitation might be written this way:

$$F_g \text{ (nt)} = G\frac{m_1 m_2}{r^2}$$

This is an excellent time to return to a point which we glossed over in a previous lesson. We now have at our command the facts we need to explain this point.

Do you recall that we emphasized over and over again the experimental fact that all bodies, regardless of their mass, have the same acceleration near the surface of the Earth, namely 9.8 m/sec^2 or 32 ft/sec^2? We do hope that this statement seemed a little incredible at the time because, after all, if a body is more massive, doesn't the Earth pull on it with more force, and shouldn't it therefore have a greater acceleration?

Now we propose to explain why all bodies have the same acceleration using Newton's Second Law of Motion plus the Law of Universal Gravitation.

As a start, suppose you select from the list below, the BEST statement of the Second Law as applied specifically to bodies in free fall.

(17)

A $a = \frac{F}{m}$

B $a = \frac{F_g}{m}$

C $g = \frac{F_g}{m}$

YOUR ANSWER --- C

No, it wouldn't remain the same. You might wonder how you could be expected to guess at a thing like this, knowing nothing at all about the laws of gravitation. Well, there are certain experiences in your background which could help you make a more "educated guess."

For example, you must have seen magnets in use at one time or another; perhaps you have played with them yourself. Magnetic force and gravitational force are similar in at least one respect: they are both forces that act over a distance. Now, in the case of magnets, you must have seen one pull a piece of iron to itself if brought sufficiently close. Then you must have noticed that, when the magnet and iron were moved far enough apart, the force was no longer great enough to move the iron.

Simply because gravitation and magnetism have certain similarities is not sufficient reason to assume that they are governed by the same natural laws. No one would expect you to make this assumption. But the similarity is there. It should encourage you to formulate a tentative hypothesis that distance of separation and resulting force might also be related in a similar fashion.

Please return to page 66 and choose another answer.

YOUR ANSWER --- C

If there is an increase in the mass of either one of the two bodies exerting gravitational force on each other, the force must change, provided that the distance between them does not change. From the equation below you should be able to recognize that a change in either m_t or m_e will cause the force to take on a new value:

$$F_g = km_tm_e$$

Please return to page 5. You can find the right answer.

YOUR ANSWER --- A

Mass and weight are entirely different concepts.

You are being misled in your thinking, perhaps, because you know that weight depends upon mass in a given location. That is, a body in Paterson, New Jersey, has a given weight; another body in Paterson, New Jersey, having twice the mass of the first object would have twice the weight.

But a dependency like that of weight on mass is not an equality. The number of apples you can buy depends upon the number of dollars you have, but apples and dollars are not the same thing.

So mass and weight cannot be equated. Later in this lesson, we will define weight very carefully so that no doubt about this should remain in your mind.

Please return to page 3. There is a better answer than the one you chose.

YOUR ANSWER --- B

A laboratory experiment can be performed in which actual measurements of the force of gravitation between relatively large masses can be taken. Of course you have not performed this experiment, but we hoped that some of your science courses in the lower grades might have given you a clue to the right answer, or perhaps that you might have read about it in popular science books.

A laboratory experiment in which the attraction between the pairs of masses in Figure 1 on page 99 is measured would disclose that the largest force would not be obtained for Case Y.

Please return to page 99. Try again.

YOUR ANSWER --- A

You are correct. Taking F_g as proportional to the product of the masses, we can write: $F_g = km_1m_2$. To compare the three cases, we could tentatively set $k = 1$ since it will have the same effect in each case. Thus, the comparative values of F_g for the three cases are:

Case X: F_g = 1 kg x 10 kg = 10 units of force

Case Y: F_g = 1 kg x 1 kg = 1 unit of force

Case Z: F_g = 10 kg x 10 kg = 100 units of force

And these values check with those of the table in Figure 2 on page 4.

Newton recognized this relationship. We shall state it as a notebook entry.

NOTEBOOK ENTRY
Lesson 9

1. The Law of Universal Gravitation

(a) If the distance between their centers is constant, the gravitational attraction between two bodies is directly proportional to the product of their masses.

$$F_g = km_1m_2$$

Please go on to page 38.

You have probably been wondering about the use of "units of force" rather than newtons in our thinking thus far. Don't worry about the units at this time; we'll clear this matter up later.

Do you have the relationship between gravitational force and mass stowed away in your mind now? Well, a little check would be useful at this time.

Suppose we have just performed the Cavendish experiment on two masses, m_1 and m_2, and have found that the force of attraction between them comes out as 16 units of force. Now imagine that we double each of the masses without altering the distance between their centers. How large would the gravitational force be now?

(4)

A 4 units.

B 32 units.

C 64 units.

YOUR ANSWER --- A

For two variables to be inversely proportional, their corresponding values must for a constant product.

In $F_g = kr^2$, we have the two variables forming a constant ratio, not a constant product because, solved for k, the relation reads $k = F_g/r^2$.

Take the time you need to study the mathematical statements. Be sure to pick the one in which you can show that the variables form a constant product.

Please return to page 6. Make your next choice more carefully.

YOUR ANSWER --- C

You neglected to include in your calculations the fact that the distance had been doubled.

If the distance had remained the same, then the value of F_g would be 12 times as large as it was initially if m_1 was multiplied by 3 (tripled) and m_2 by 4 (quadrupled).

But you were told that r had doubled, too.

So, return to page 97 and select the right answer.

YOUR ANSWER --- A

The ratio of the forces on Io and Callisto does not fall into this range. You have, therefore, made an error in order of magnitude.

You must go back and find your error by checking all of the operations you performed. In a problem of this type, there are three usual sources of error:

(1) Forgetting to square the distances.
(2) Errors in handling powers of 10.
(3) Just plain arithmetic errors (such as division of fractions).

Please return to page 8 and make another selection.

YOUR ANSWER --- B

You should remember how we obtained the equivalence of the newton from the Second Law.

$$F = ma$$

$$F = kg \times \frac{m}{sec^2} = \frac{kg\text{-}m}{sec^2}$$

Then we merely began to use the name newton for the $\frac{kg\text{-}m}{sec^2}$. Thus by definition,

$$1 \text{ newton} = 1\frac{kg\text{-}m}{sec^2}$$

If the newton is defined this way, how can it possibly be the same as 1 kg^2/m^2? It would be just as logical (or, better, illogical) to say that 1 lb is the equivalent of 1 apple per $orange^2$.....or that 1 ft is the equivalent of 1 ton/lb^2. We can't have different definitions for the same thing when we use it in different places. There is absolutely nothing you can do to cause two different units to take on the same meaning, so don't try it.

But there is something that we can do to get out of this difficulty. Try to see what that way must be!

Please return to page 106 and try again.

YOUR ANSWER --- B

You seem to have become mixed up by the universal equation and the one in which we substituted units. No proper arithmetic manipulations could ever lead to a simplification like this one. In performing simplifications, you must observe the rules of ordinary arithmetic. You can do it if you take your time.

Try again. Return to page 74 and make another selection _after_ you have worked it over once more.

YOUR ANSWER --- A

This answer is incorrect.

To find the right answer, it is necessar substitute unity for each of the independent variables (m_1, m_2, and r) as .l as the constant k.

You will find that the answer is not 0.1 nt.

Please return to page 13. Make the suggested substitutions. What do you get for F_g?

YOUR ANSWER --- A

This answer contains two mistakes.

First, the unit is wrong. Check your work and find the right unit.

Second, you neglected to take into account that the distance is 10 centimeters, not 10 meters.

Make the necessary corrections; then please return to page 76 and choose the right answer.

YOUR ANSWER --- B

This expression is a perfectly good statement of the Second Law, but it can be made even better by using another letter for the acceleration. Do you recall that we use the letter _g_ rather than _a_ to indicate acceleration in free fall?

Please return to page 32. You can choose the best equation now.

YOUR ANSWER --- B

Can't be done! At least not according to the standard rules of arithmetic. You have no justification for such an operation!

There's an easy, legitimate step you can perform to simplify the equation.

Please return to page 111 and try again.

YOUR ANSWER --- A
(PAGE 78)

$$g = G \frac{m_e}{r^2}$$

Look at the equation above carefully; then read the statement even more carefully. It sounds good, but something has been omitted. What is the exponent of the distance symbol?

Please turn to page 79 after you have discovered what is missing.

YOUR ANSWER --- B

Although your coefficients and exponential numbers are correctly associated with each other, you have an inversion in the substitution. You can find this error.

Please return to page 113 and select the right substitution.

YOUR ANSWER --- C

We are going to say that this answer is wrong because of a certain mistaken idea it has.

The symbol "G" stands for the universal constant of gravitation. It always has the same value, throughout the entire universe--namely, 6.67×10^{-11} $m^3/kg\text{-}sec^2$. Remember that the need for G arose in developing the gravitational equation only because a proportionality constant was required that (a) had the right units to make our answer come out in familiar force units and (b) had the right magnitude to make our answer come out in newtons.

Thus if you accept G_m, this implies that the constant has one special value on Mars and other values elsewhere. This is not true; hence the equation should not be accepted in this form.

Please return to page 115. The right answer should now stand out clearly.

YOUR ANSWER --- A

In collecting terms, you should have arrived at something like this:

$$w = \frac{6.7 \times 6.4}{(3.4)^2} \times \frac{10^{-11} \times 10^2 \times 10^{23}}{10^{12}}$$

In the exponential portion of this expression, then, the numerator turns out to be 10^{14}. For you to have obtained the answer you selected, you would have had to omit taking the 10^2 into account. This would make the numerator 10^{12} which would be exactly canceled by the denominator to yield 10^0.

Please return to page 21. You should be able to pick out the right answer at once.

YOUR ANSWER --- B

There is a serious error here.

You have placed a "2" before the "r^2." What does this mean? Do you think it expresses the fact that the weight of the rocket must decrease to half that at sea level?

No. The "2" before the "r^2" says that the value of r^2 has been doubled. There is nothing in the statement of the problem to justify this assumption. Just because the weight is going to go down by a factor of 2 is no reason to assume that the distance between the rocket and the center of the Earth will double.

Perhaps you were thinking of $m_r/2$, but this would not be right either because the mass of the rocket doesn't change (for purposes of this problem). Actually mass is decreased as fuel is burned, but we are looking for effects of gravitational pull only. The decrease of fuel is another problem.

Please return to page 84 and select the correct answer.

YOUR ANSWER --- A

An excellent way to tell if any quantity remains unchanged from one equation to another is to look for a repetition of its symbol in both equations. One of the three quantities in the answer above is not repeated in both equations; hence it is changing. A quantity that changes does not remain constant.

Please return to page 23 and select another answer.

YOUR ANSWER --- B

You are correct. To two significant figures, the force acting on Io is about 16 times as great as that acting on Callisto. The solution follows:

Mass of Io = m_i = 1.0 unit of mass.
Mass of Callisto = m_c = 1.3 units of mass, comparatively.
Distance of Io = 2.6×10^5 mi. This is r_i.
Distance of Callisto = 1.2×10^6 mi. This is r_c.

Force acting on Io = $F_i = k\frac{m_i m_J}{r_c{}^2} = k\frac{1.0 \times m_J}{(2.6 \times 10^5)^2} = \frac{km_J}{6.8 \times 10^{10}}$

Force acting on Callisto = $F_c = k\frac{m_c m_J}{r_c{}^2} = k\frac{1.3 \times m_J}{(1.2 \times 10^6)^2} = \frac{1.3\ km_J}{1.4 \times 10^{12}}$

Ratio of forces: $\frac{F_i}{F_c} = \frac{\frac{km_J}{6.8 \times 10^{10}}}{\frac{1.3\ km_J}{1.4 \times 10^{12}}} = \frac{\cancel{km_J}}{6.8 \times 10^{10}} \times \frac{1.4 \times 10^{12}}{1.3\ \cancel{km_J}}$

$$\frac{F_i}{F_c} = \frac{1.4}{1.3 \times 6.8} \times 10^2 = \underline{16\ \text{(answer)}}$$

That was a little tougher than usual, wasn't it? But when you come right down to the difficulty, you can realize that the only tedious part was the arithmetic. The principles used in solving this problem are no different from those used on the simpler ones we solved before.

Please go on to page 55.

Not once have we involved the force unit that would have to appear in any solution of

$$F_g = k\frac{m_1 m_2}{r^2}$$

Of course, we had our reasons, but we can't avoid this question forever! To be consistent with the MKS system, what would we like this force unit to be?

(11)

A The pound.

B The kilogram.

C The newton.

YOUR ANSWER --- D

You are correct. The force would be:

$$F_g = 1\frac{1 \times 1}{(1)^2} = 1 \text{ nt}$$

In obtaining a force of 1 nt, you will remember that we assumed k = 1 $m^3/kg\text{-}sec^2$. This assumption was entirely unwarranted and, as a matter of fact, is decidedly incorrect. Once we have defined the newton in terms of the Second Law (1 nt = 1 kg x 1 m/sec^2) we are committed to this value. We are no longer free to define it in terms of other variables, such as those in the Law of Universal Gravitation. Thus, two 1 kg masses with 1 m between their centers do not exert a force of 1 nt on each other. This means that k does not equal 1 $m^3/kg\text{-}sec^2$.

How, then, does the physicist determine the true numerical value of the constant of proportionality in the gravitation law? There is only one way. It must be obtained from experimental data. Henry Cavendish was the first to make the necessary measurements.

Please go on to page 57 to see how he did it.

Before continuing, please turn to page 123 in the blue appendix.

A simplified diagram of his apparatus is shown in Figure 4. Two large masses, M, were mounted rigidly in a closed chamber near two smaller masses, m. The small masses were suspended from a thin flexible wire.

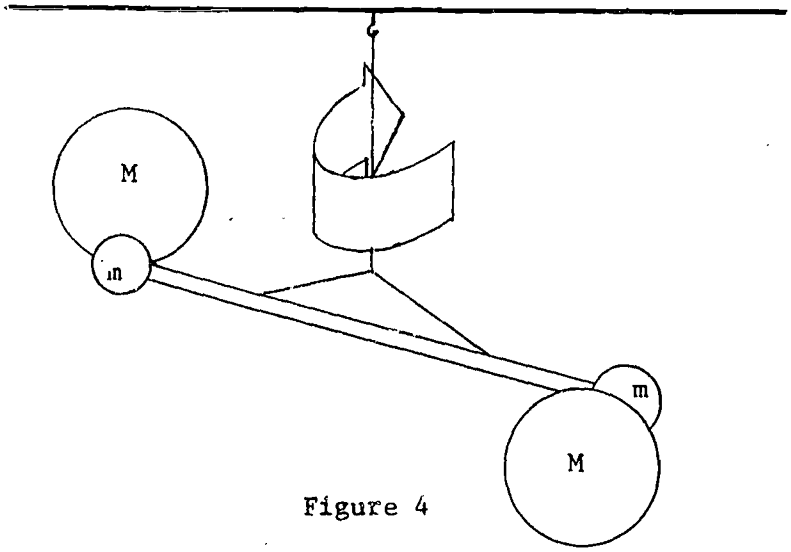

Figure 4

The small masses were attracted toward the larger ones and, despite the tiny gravitational force, the suspension wire twisted slightly, displacing the small masses by a measurable amount. Cavendish recorded the amount of twist; then he moved both large masses to a symmetrical position on the opposite sides of the small masses. He again recorded the amount of twist. From a knowledge of the forces required to twist that particular suspension wire known amounts, he was able to calculate the force of gravitation. He knew each of the masses (m and M) accurately, and he knew the distance between their centers to a high degree of precision. The amount of twist told him the magnitude of the attractive force between m and M. That left only one unknown, namely, ________. WHAT IS THE MISSING SYMBOL?

(15)

A r

B k

C F_g

D m_2

YOUR ANSWER --- D

These are two entirely different masses. They cannot be combined in symbol form by multiplying them and then calling the product the "square of" some mass, m.

The actual legitimate operation is staring you in the face!

Please return to page 111 and determine what it is.

YOUR ANSWER --- D

If you know only G and r, the equation $g = G \times m_e/r^2$ still has two unknowns, m_e and g. As you well know, an equation containing two unknowns cannot be solved for either of them. To find g without experimentation, you would also have to know the mass of the Earth, m_e.

Read the other answers carefully; think about them. What was the point we set out to prove?

Please return to page 78 and select the right answer.

YOUR ANSWER --- A

You didn't solve it correctly.

It would be best if you tried this problem again with no help from us. So review your steps and try to find your mistake.

Please return to page 24, please, and make another selection.

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YOUR ANSWER --- A

You are correct. That is:

Initially: $F_g = k\frac{m_1 m_2}{r^2}$

After change: $F_g = k\frac{(3m_1)(4m_2)}{(2r)^2} = k\frac{12m_1 m_2}{4r^2}$

New force: $F_g = 3k\frac{m_1 m_2}{r^2}$

And the new force is three times as large as the initial force.

To see how the Law of Universal Gravitation applies in astronomy, let's investigate it in reference to the two satellites of Jupiter, namely Io and Callisto. (See Figure 3.)

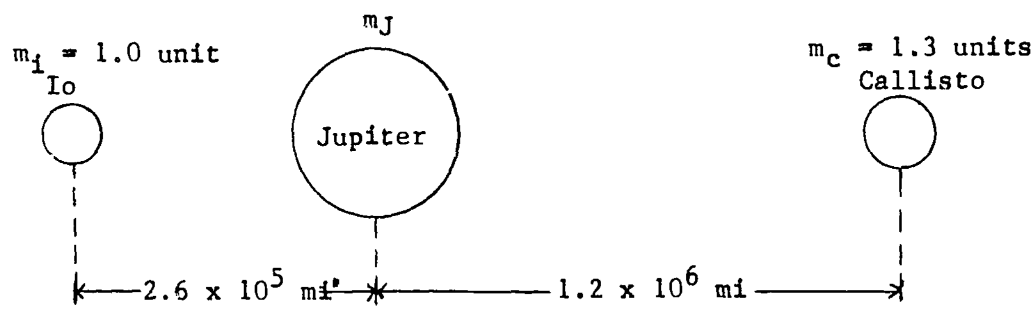

Figure 3

Note first how the masses are given. The mass of Jupiter is designated simply as m_J; as you will see, the actual mass in kilograms will not be required. In addition, the mass of Io (m_i) is stated as <u>1 unit</u> while that of Callisto is given as m_c = <u>1.3 units</u>. All this means is that Callisto has 1.3 times the mass of Io. The actual masses in kilograms will not be needed in solving the problem we intend to present.

Now observe the distances of the satellites from their primary, Jupiter. Study these figures; then answer this question.

Which satellite experiences a greater gravitational force?

(9)

A Io.

B Callisto.

YOUR ANSWER --- B

You are correct. From the following equation:

$$F_g = km_t m_e \qquad \text{(present circumstances)}$$

we can obtain this:

$$F_g = k(10m_t)m_e \qquad \text{(increased mass of Tiros)}$$

$$= 10km_t m_e$$

Since $10km_t m_e$ is 10 times as large as $km_t m_e$, then the new gravitational force would be 10 times the original value.

We'll leave the subject of mass, for the moment, and turn to the question of separation distance between the centers of the bodies and the effect of this distance on gravitational force. Newton arrived at a relationship between distance and force by observing the motions of the moon and the planets; Henry Cavendish corroborated Newton's equation experimentally. Let's see where some independent thinking on your part would take you in this regard.

Let's say that you have measured the gravitational force between two known masses separated by a known distance. Now assume that the bodies are moved apart, increasing the distance between their centers. Would you then expect the force of gravitation to increase, decrease, or remain the same?

(6)

A Increase.

B Decrease.

C Remain the same.

YOUR ANSWER --- B

The amount of space occupied by a body does not determine its mass.

A familiar example will point this out very quickly. Say you determine the mass of a deflated rubber balloon by any method you wish. You might weigh it first; then you might look up the value of g for your location; and finally you might use the equation $w = mg$ to determine the mass by solving for m and substituting your weight measurement and the value of g.

$$m = \frac{w}{g}$$

Now you inflate the balloon to a large volume. The space it occupies will now be 20 to 1,000 times greater than it was before, depending upon the kind of balloon it is. The weight of the inflated balloon might be slightly greater than it was before because of the enclosed air but it certainly would not be even twice or three times as great, let alone 20 to 1,000 times greater. You know this from experience.

Therefore, although you have increased the space occupied many times over, the weight increase is certainly not in the same proportion, so the mass increase cannot be in the same proportion either.

It would be wise if you were to divorce mass from volume in your mind from now on. Only under certain circumstances are they interdependent; almost all of the time they are not.

Please return to page 3 and choose another answer.

YOUR ANSWER --- A

The only way to determine which of the three cases is the one where gravitational force is the greatest is to perform a laboratory experiment using the masses shown in which you measure the actual attractive force. In asking you to select the proper answer, we were more or less depending on your previous reading and, perhaps, on your past studies in the lower grades.

It is possible to measure the gravitational attraction between laboratory masses, as you will see later. But such a measurement performed on the three cases in Figure 1 on page 99 does not give the answer you selected

Please return to page 99 and choose another answer.

YOUR ANSWER --- B

If F_g were directly proportional to the sum of the masses, we could write: $F_g = k(m_1 + m_2)$. To compare the three cases, we could then set k = 1 since it will have the same effect in each situation. Next, we can determine the value of F_g in a comparative sense in this manner:

Case X: F_g = 1 kg + 10 kg = 11 units of force

Case Y: F_g = 1 kg + 1 kg = 2 units of force

Case Z: F_g = 10 kg + 10 kg = 20 units of force

In the chart of Figure 2 on page 4, the values of F_g are, respectively, 10 units, 1 unit, and 100 units. Surely you can see that the comparative values are completely different for the Cavendish measurements than they are when obtained from the sums of the masses.

Hence, F_g is <u>not</u> proportional to the <u>sum</u> of the masses.

Please return to page 4. Using the method outlined above, you should be able to arrive at the right answer now.

YOUR ANSWER --- A

The force of gravitation is directly proportional to the product of the masses. Your answer indicates that you are handling these quantities as though there were an inverse proportion between them.

You should do it this way:

In general:	$F_g = km_1m_2$
Originally, before doubling:	$16 = km_1m_2$
After doubling each mass:	$F_g = k(2m_1)(2m_2) = 4km_1m_2$
So the new force is:	$F_g = 4km_1m_2$

From the second step we know that km_1m_2 originally had a value of 16 units. So we can substitute 16 units of force for the factor km_1m_2 in the last expression. Thus we have:

$$F_g = 4(16)$$

This does not give you an answer of 4 units of force, does it?

Please return to page 38. You should be able to pick the right answer now.

YOUR ANSWER --- B

You weren't careful. The expression you selected states that F_g is inversely proportional to the separation distance. We want the expression which states that F_g is inversely proportional to the square of the separation distance.

Please return to page 6 and make your next choice carefully.

YOUR ANSWER --- C

The ratio of the forces on Io and Callisto does not fall into this range. You have, therefore, made an error in order of magnitude.

You must go back and find your error by checking all of the operations you performed. In a problem of this type, there are three common sources of error:

(1) Forgetting to square the distances.
(2) Errors in handling powers of 10.
(3) Just plain arithmetic errors (such as division of fractions).

Please return to page 8. After working the problem again, you should be able to select the right order of magnitude.

YOUR ANSWER --- B

That's not a thoughtful answer.

The kilogram measures only one thing: mass.

We do not use it to measure force, although some attempts have been made in the past to establish the kgf (kilogram-force) as a standard unit. In our endeavor to remain consistent at all times, what should always appear in our discussions as the MKS force unit?

Please return to page 55. Make another selection.

YOUR ANSWER --- C

You are correct. If this was your first answer choice, you are doing very, very well. The constant in the Law of Universal Gravitation is not a pure number; it has units of its own and these units must be assigned to it by us in such a way that F_g will come out in newtons. The easiest way to determine the units that must be assigned to k in order to make F_g come out in newtons is to solve the gravitation equation for k, and then substitute MKS units for the other quantities. Thus:

The Law: $$F_g = k\frac{m_1 m_2}{r^2}$$

Then, solved for k: $$k = \frac{Fr^2}{m_1 m_2}$$

Remembering that the newton is a $\frac{\text{kg-m}}{\text{sec}^2}$, we may then substitute and obtain:

$$k = \frac{\frac{\text{kg-m}}{\text{sec}^2} \times \text{m}^2}{\text{kg}^2}$$

Whew! That's a complicated-looking expression. But it can be simplified. Try to do it. Then choose one of the expressions below as the correct simplification.

(13)

A $\frac{\text{m}^2}{\text{sec}^2}$

B $\frac{\text{kg-sec}^2}{\text{m}^2}$

C $\frac{\text{m}^3}{\text{kg-sec}^2}$

YOUR ANSWER --- B

This answer is particularly poor because it contains two errors. First, it is numerically wrong. Second--and much worse--you accepted the kilogram as a force unit. The kilogram is used only as a mass unit!

Please return to page 13. Select your answer more carefully this time.

YOUR ANSWER --- B

You are correct.

We'll take some representative figures. In a modern laboratory where the Cavendish experiment is being repreated it is found that a 1.00 kg mass attracts a 10.0 kg mass with a force of 0.667×10^{-7} newtons when the distance between the centers of the two masses is 10.0 cm.

To determine the numerical value of k, we then solve the gravitational equation for k, substitute the known quantities, and there it is.

Suppose you do the job. What is the value of k with units? Which one of the quantities given below is the correct one?

(16)

A $0.667 \times 10^{-6} \frac{m^3}{kg\text{-}sec}$

B $0.667 \times 10^{-8} \frac{m^3}{kg\text{-}sec^2}$

C $0.667 \times 10^{-9} \frac{m^2}{kg\text{-}sec^3}$

D $0.667 \times 10^{-10} \frac{m^3}{kg\text{-}sec^2}$

YOUR ANSWER --- C

You are correct. This is a good way to write it because it shows that the acceleration is due to gravity (g rather than a) and that the force acting on the mass is the gravitational force (F_g) rather than any other force.

Let's bear this in mind: we want to show that the mass of the falling body (m) has absolutely no effect on its acceleration. To keep our symbols very clear, suppose we call the mass of the falling object m_o rather than just m alone. So then, the acceleration of this object is given by the relation:

$$g = \frac{F_g}{m_o}$$

But we know an equation that will enable us to write F_g in terms of the mass of the object (m_o), the mass of the Earth (m_E), the universal constant of gravitation (G), and the distance between the center of the object and the center of the Earth.

All right. Write F_g in terms of these quantities, using the symbols given in the paragraph above. Then turn to page 111.

YOUR ANSWER --- A

Sure, that is correct. When the m_o terms are canceled, the simplified expression becomes:

$$g = G\frac{m_e}{r^2}$$

Are you surprised at this result? You knew all along that something like this was going to happen. The mass of the object in free fall (m_o) has dropped out of the equation altogether.

But what does this expression for g mean? Pick out the one statement below that tells us what we want to know.

(19)

A The acceleration in free fall is directly proportional to the mass of the Earth and inversely proportional to the distance between the center of the Earth and the center of the falling body.

B The magnitude of g--acceleration in free fall--is the same for any object no matter where it may be.

C The acceleration of a freely falling body is independent of the mass of the body.

D We could compute the value of g without experimentation if we are merely told the value of G and the radius of the Earth (r).

CORRECT ANSWER: The words "square of" have been omitted before the word "distance." That is, the latter part of the statement should read "...and inversely proportional to the square of the distance between the center of the Earth and the center of the falling body."

This kind of mistake is inevitable if you don't read carefully.

Please turn to page 78.

YOUR ANSWER --- A

You are correct. As indicated by $g = G \times m_e/r^2$, g is inversely proportional to the square of the distance between centers. By rising to 4,000 miles, the distance to the center of the Earth has been doubled since the radius of the Earth is also about 4,000 miles. Hence, the value of g would be cut down to ¼ of its surface value. Since the surface value is 32 ft/sec^2, then the new value would be 32/4 ft/sec^2 or 8 ft/sec^2.

Think now! What was it we set out to prove about the effect of mass of a freely falling body on its acceleration? Let's go back and take another look at the question we are dealing with.

Please turn to page 78.

YOUR ANSWER --- A

Check the substitution very carefully. Your coefficients are correctly positioned, but you have an error in placement in the powers of 10. You can find the error.

Please return to page 113 and select the right substitution.

YOUR ANSWER --- A

This is a statement of Newton's Second Law. In it, $\underline{w}$ is the weight of the person, m_p is the mass of the person, but what is $\underline{g}$? Normally, $\underline{g}$ stands for the gravitational acceleration near the Earth's surface, 9.80 m/sec^2.

Now, if you meant the gravitational acceleration near the surface of Mars, you must identify the $\underline{g}$ as g' or g_m or some other clearly defined symbol. Then, the correct equation for the weight on Mars would be:

$$w = m_p g_m$$

The trouble is, of course, that you don't know g_m, the gravitational acceleration on Mars, so you couldn't use this equation to determine the weight of the 100 kg man.

Please return to page 115. Select a better answer.

YOUR ANSWER --- C

In collecting terms, you should have arrived at something like this:

$$w = \frac{6.7 \times 6.4}{(3.4)^2} \times \frac{10^{-11} \times 10^{2} \times 10^{23}}{(10^{6})^{2}}$$

If you forget to square the factor in the denominator, then you would get an answer of 10^8 for the exponential portion of the weight. So this must be what you did.

Work out the exponential term again, remembering to square 10^6. We're sure you'll hit it this time.

Please return to page 21.

CORRECT ANSWER: A 220 lb man would weigh about 82 lb on Mars.

Here is another interesting type of problem. Now that man has fully committed himself to space travel, he is eager to get all kinds of information about the vehicles that he blasts from the Earth. For example, if a rocket is fired straight upward, the distance between its center and the Earth's center increases as long as it continues to move away from the Earth.

We'd be interested in knowing, say, the altitude to which a rocket must be carried by its motors for its weight to decrease to half the value it has at sea level.

How does one tackle such a problem? As a start, let's write the equation for the weight of the rocket at sea level:

$$w = G\frac{m_r m_e}{(4,000)^2}$$

in which m_r = the mass of the rocket and "4,000" is the distance between the center of the rocket and the center of the Earth (which, of course, is merely the radius of the Earth in miles.)

Next let us write a similar gravitational equation for the condition in which the weight of the rocket is half-normal.

Which one of the following is the correct equation?

(24)

A $\frac{w}{2} = G\frac{m_r m_e}{r^2}$ where r is the new distance from the rocket to the center of the Earth.

B $w = G\frac{m_r m_e}{2r^2}$ where r is the new distance from the rocket to the center of the Earth.

YOUR ANSWER --- B

How do you determine if a given quantity remains unchanged from one equation to another? You look for the symbol of this quantity to be repeated in the same form in both equations.

Now if you do this for the three quantities above, you will see that they are not all repeated as you go from one equation to the other; hence at least one of these quantities changes for the new situation. A quantity that changes does not remain constant.

Please return to page 23. You should be able to choose the correct answer now.

YOUR ANSWER --- B

No, this is not correct. The error is a rather common one committed by students in dividing fractions. On the left side:

$$\frac{w}{\frac{w}{2}} = \frac{1}{\frac{1}{2}} \quad \text{(After canceling w's.)}$$

But 1 divided by $\frac{1}{2}$ does not equal $\frac{1}{2}$, does it?

Please return to page 117 and note that the other answer is correct.

YOUR ANSWER --- B

This is not the correct solution.

It would be best for you to solve this problem with no help from us. Review the steps that brought you to the above solution and try to find the error.

Please return to page 24 and select another answer.

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YOUR ANSWER --- D

You need to review the meanings of the symbols in the Law of Universal Gravitation.

$$F_g = \frac{km_1m_2}{r^2}$$

We said that Cavendish knew the masses M and m accurately. In this case, we have used the letter M for m_2, so it is known.

Think over the meaning of each symbol a little more carefully. Then return to page 57 and try again.

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YOUR ANSWER --- C

This solution is not correct.

It would be best for you to solve this problem with no help from us. Review the steps that carried you to the solution you selected; try to find the mistake you made.

Please return to page 24 and find the right answer.

CORRECT ANSWER: The combined proportion should appear like this:

$$F_g = k\frac{m_1 m_2}{r^2}$$

Two proportions involving the same dependent variable may be combined into a single proportion merely by putting them together in the form shown above. We have not presented a rigorous mathematical proof that this "putting together" process is correct (although such a proof <u>does exist</u>), but we know that it <u>works</u>. It is easily tested. Let's run through it together.

If the distance between the centers of two gravitating bodies is not allowed to change, then <u>r</u> is constant. This means that r^2 is also constant so that k/r^2 is constant, too. Let's replace k/r^2 with K to signify that this entire fraction remains the same. This gives us: $F_g = Km_1m_2$ which is nothing more than a symbolic statement of the first part of the Law of Universal Gravitation. (See notebook entry 1(a) for this lesson.) Thus far, then, the combined proportion works.

Next, we propose to let the distance between centers vary, while the masses are held constant. In that case, the product m_1m_2 is constant, making the whole product km_1m_2 a constant, too. Again, we can replace km_1m_2 with K to signify its unchanging nature and write:

$$F_g = \frac{K}{r^2}$$

This is, of course, the second part of the Law of Universal Gravitation. (See notebook entry 1(b).)

So, you see the expression $F_g = k\frac{m_1 m_2}{r^2}$ is a <u>complete</u> statement of the law.

Please turn to page 122 in the blue appendix.

Before going ahead, be sure to complete notebook entry 1(c) with:

$$F_g = k\frac{m_1m_2}{r^2}$$

You are now equipped to handle so-called functional problems in the Law of Universal Gravitation. Such problems do not involve units; they merely require you to state the factor by which the dependent variable changes (F_g in this case) when you change one or more of the independent variables (m_1, m_2, or r) in a stated way.

For example, suppose you double m_1, m_2, and r. Yes, all of them. What change would that make in the force of gravitation between the two bodies? The solution follows:

Initially: $$F_g = k\frac{m_1m_2}{r^2}$$

After Doubling: $$F_g = k\frac{(2m_1)(2m_2)}{(2r)^2} = k\frac{4m_1m_2}{4r^2}$$

(NOTE: In doubling the distance, the "2" must be placed inside the squared parenthesis! You are doubling the distance; you are not doubling the distance squared!)

Note that the 4's cancel so that the new force is:

$$F_g = k\frac{m_1m_2}{r^2}$$

This is exactly the same as it was initially. In other words, doubling all the independent variables leaves the force of gravitation unchanged.

Please go on to page 97.

Now you try this one: What happens to F_g if the mass of one body is tripled and the other mass is quadrupled while the distance is doubled?

The value of F_g will go up by what factor?

(8)

A 3

B 6

C 12

YOUR ANSWER --- A

No, this answer is wrong. But how could you have made a more "educated guess" than this, assuming that you know nothing at all about gravitation? There is a way; admittedly, it's not especially scientific, but very often small bits of apparently unrelated knowledge lead to better guesses.

You must have played with magnets at one time or another. Magnetic force acts over a distance and is therefore quite similar in this respect to gravitational force. You know that a magnet will draw a piece of iron to itself if it is brought sufficiently close to it. Now move the same magnet further away from the same piece of iron, and you find that it can no longer exert enough force to make the iron move.

Indeed you have no right to assume that magnetism and gravitation follow the same physical laws, so you cannot say that everything which is true about magnetism must also be true about gravitation. But their similarity is strong enough to give you a jumping off place for your guesses.

Please return to page 66 and make another selection.

YOUR ANSWER --- C

You are correct. The mass of a body depends upon the quantity of matter in it: the number and kind of molecules and how closely they are packed.

We hope you remember that we have considered the definition of mass in terms of "quantity" of matter as a very weak one from the beginning. The word quantity is a vague and not particularly meaningful term. In our study of the laws of motion, we found that it was highly satisfactory to relate mass to inertia. That is, a body of small mass is accelerated by a small force, while a body of large mass requires a correspondingly larger force to give it the same acceleration.

Now in considering the gravitational force between two objects, we must recognize that we are now dealing with two masses rather than with one as we did in applying the laws of motion.

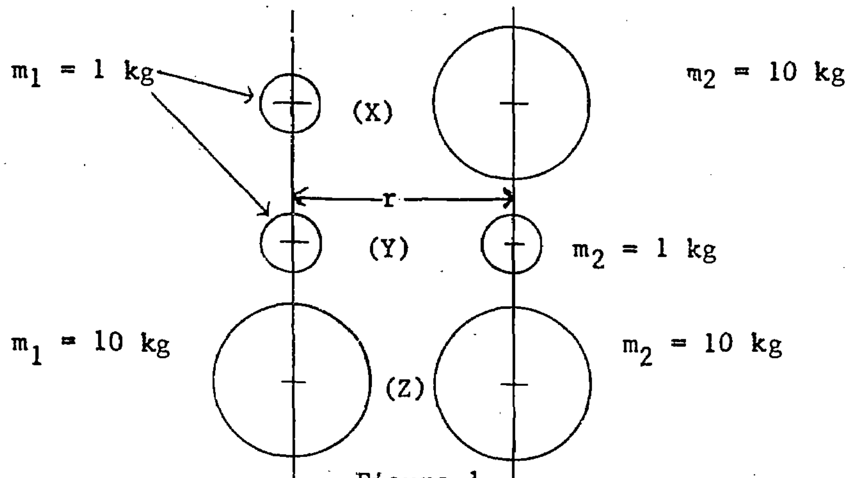

Figure 1

Refer to Figure 1. X, Y, and Z represent three different cases. In each case, the distance r between the centers of the two bodies is the same, but the combination of masses is different. There is a mutual gravitational attraction between paired masses in each case. In which one of the three cases do you think the attractive force is greatest?

(2)

A Case X.

B Case Y.

C Case Z.

YOUR ANSWER --- A

We didn't say that the mass of the Earth was also increased 10 times! Only that of Tiros II has been increased tenfold.

In looking at the proportion for this problem, you would say:

$$\begin{array}{lll} \text{(Present)} & F_g = km_t m_e & \left\{ \begin{array}{l} \text{where } m_t = \text{Tiros II's mass} \\ \text{and } m_e = \text{Earth's mass} \end{array} \right] \\ \text{(Imaginary)} & F_g = k(10m_t)m_e & \end{array}$$

We won't go any further than this here. Merely by looking at the two proportions, you should be able to see your error.

Please return to page 5. Select a better answer.

YOUR ANSWER --- C

If F_g were directly proportional to the quotient of the masses, we could write:

$$F_g = k\frac{m_2}{m_1}$$

To compare the three cases, we could then set k = 1 since it will have the same effect in each situation. We could next determine the value of F_g in a comparative sense in this manner:

$$\text{Case X:}\quad F_g = \frac{10\ \text{kg}}{1\ \text{kg}} = 10 \text{ units of force}$$

$$\text{Case Y:}\quad F_g = \frac{1\ \text{kg}}{1\ \text{kg}} = 1 \text{ unit of force}$$

$$\text{Case Z:}\quad F_g = \frac{10\ \text{kg}}{10\ \text{kg}} = 1 \text{ unit of force}$$

The Cavendish experiment values of F_g, however, are respectively, 10 units, 1 unit, and 100 units. Surely you can see that the comparative values of the Cavendish measurements are vastly different from those obtained by assuming that F_g is proportional to the quotient of the masses.

Please return to page 4. Using the method outlined above, you should be able to figure out the right answer.

YOUR ANSWER --- B

You are not handling the proportion correctly.

Look here:

In general: $F_g = km_1m_2$

Originally: $16 = km_1m_2$

Now we double each mass: $F_g = k(2m_1)(2m_2) = 4km_1m_2$

So the new force is: $F_g = 4km_1m_2$

From the second step we know that km_1m_2 originally had a value of 16 units. So we can substitute 16 units of force for km_1m_2 in the last expression. Thus:

$$F_g = 4(16)$$

This does not give you an answer of 32, does it?

Please return to page 38. You can now select the right answer.

YOUR ANSWER --- C

You are supposed to choose the expression which mathematically states that the force of gravitation is inversely proportional to the square of the separation distance. That is, the r should be squared. Why did you square the constant k? This would still be a constant and would not show a relation to change in separation. Furthermore, the k is in the numerator, making F increase as k increases.

Please return to page 6. Make a more thoughtful selection.

YOUR ANSWER --- B

No. It's the other way around.

Look at the figures again.

	Mass	Distance
Io	1.0 unit	2.6×10^5 mi
Callisto	1.3 units	1.2×10^6 mi

Callisto has a larger mass than Io. This in itself would tend to cause the gravitational pull of Jupiter on Callisto to be about 1.3 times as great as that on Io.

But, compare the distances. Io is 4.5 times closer to its primary than Callisto is. Since gravitational force varies inversely as the square of the distance, this would tend to produce a force on Io that would be $(4.5)^2$ or 20 times as great as it is on Callisto. Naturally, this more than overcomes the effect of the greater mass of Callisto.

Clearly, then, you will want to choose the alternative answer to the original question. Please return to page 65 and do so.

YOUR ANSWER --- D

Yes it is!

One of the ranges given is correct for the ratio of F_g acting on Io to F_g acting on Callisto. You have made a seriou ror in order of magnitude if you cannot find the range in which your answe ongs among those listed.

Possibly you committed one or more of the following mistakes:

(1) Did you forget to square the distances?
(2) Did you handle the powers of 10 correctly.
(3) Were you careful with your arithmetic, especially in the division of fractions?

Please return to page 8. After working through the problem again, you should be able to choose the right order of magnitude.

YOUR ANSWER --- C

Well, then, let's go back to the Law of Universal Gravitation in its mathematical form:

$$F_g = k\frac{m_1 m_2}{r^2}$$

We shall first view the k as a pure number, substitute units on both sides, and then see whether this assumption is correct. Remember, a pure number has no units at all. If we do this:

Since

F_g is to be in newtons
m_1 and m_2 in kilograms
r in meters

we have: newtons $= k\frac{\text{kg} \times \text{kg}}{\text{m}^2}$

But we know that a newton is the equivalent of a $\frac{\text{kg-m}}{\text{sec}^2}$; so:

$$\frac{\text{kg-m}}{\text{sec}^2} = k\frac{\text{kg}^2}{\text{m}^2}$$

Well! The units now shown on the opposite sides of the equation certainly are not equivalent, are they?

Think now! Which one of the statements below is the best explanation of this apparent dilemma?

(12)

A There is nothing we can do to make F_g come out in newtons.

B Obviously, $1\frac{\text{kg}^2}{\text{m}^2}$ is the same as 1 newton.

C Our assumption was not correct; we cannot assume k to be a pure number.

YOUR ANSWER --- A

If you could cancel the kg-m in the numerator against the kg^2 in the lower denominator, then the simplification would be correct. But, unfortunately, this cancellation is improper; hence this simplified expression is wrong.

Take your time. Observe the rules of simple arithmetic. You can do it.

Please return to page 74 and try again.

YOUR ANSWER --- C

You know better than that. The kilogram is a mass unit, not a force unit.

Please return to page 13. Choose another answer.

YOUR ANSWER --- C

You need to review the meanings of the symbols in the Law of Universal Gravitation:

$$F_g = k\frac{m_1m_2}{r^2}$$

F_g is the gravitational force or the force of attraction between the masses. Thus, if F_g was measured, it was known, not unknown.

Please, try a bit harder. Return to page 57 and make another selection .

YOUR ANSWER --- C

There are two errors in this answer.

The unit is incorrect. Check your previous work and find the right unit.

Also, the answer is numerically wrong because you mishandled your exponents.

Go through the solution once more. Then return to page 76 and select a better answer.

CORRECT ANSWER: F_g should be written in terms of m_o, m_E, G, and r as follows:

$$F_g = G\frac{m_o m_e}{r^2}$$

Did you get it right? We hope so!

Now we have the Second Law:

$$g = \frac{F_g}{m_o}$$

Since we also have the equivalent of F_g in the form of the equation in the answer above, we can substitute this equivalent for F_g in the Second Law equation and obtain:

$$g = \frac{G\frac{m_o m_e}{r^2}}{m_o} \quad \text{(This replaces } F_g\text{.)}$$

Don't let the complex appearance of this equation frighten you. Look at it calmly. You should notice in a few seconds that there is a way to simplify it considerably by means of a single operation. What is this operation?

(18)

A Cancel the m_o in the numerator by the m_o in the denominator.

B Move the r^2 into the numerator.

C Cancel g against G, dropping out these terms.

D Multiply m_o by m_e, thus obtaining m^2.

YOUR ANSWER --- B

The location of an object with respect to the planet whose mass causes the gravitational force determines the value of r. If an object is close to the Earth, the distance of separation between its center and that of the Earth is about 4,000 miles, equal to the radius of the Earth. But if the object is carried to some distance away from the Earth, r increases causing g to decrease. For example, if a rocket is fired to an altitude of 4,000 miles above the surface of the Earth, its new distance of separation from the Earth's center is 8,000 miles. The value of r has therefore doubled. What would be the new value of g? Select one of the answers below:

(20)

A The new value of g would be about 8 ft/sec^2.

B The new value of g would be about 16 ft/sec^2.

CORRECT ANSWER: Solved for m_e, the equation $g = G\frac{m_e}{r^2}$ becomes:

$$m_e = \frac{gr^2}{G}$$

The values of the known quantities on the right side of the equation are (all to two significant figures):

$g = 9.8$ m/sec^2
$r = 4.0 \times 10^3$ miles (radius of Earth, average)
$G = 6.7 \times 10^{-11}$ m^3/kg-sec^2

To clear up the inconsistency in the units, we'll convert the radius of the Earth to meters for you:

$$r = 4.0 \times 10^3 \text{ mi} \times 5.28 \times 10^3 \text{ ft/mi} \times 1/3.28 \text{ m/ft}$$
$$= 6.44 \times 10^6 \text{ meters}$$

Since we will need to square r, let's do that right away:

$$r^2 = (6.44 \times 10^6)^2 = 4.14 \times 10^{13} \text{ meters}^2$$

We're now ready to substitute. You do this part of it; then choose the correct substitution from those below.

(21)

A $m_e = \frac{9.8 \times 4.1 \times 10^{-11}}{6.7 \times 10^{13}}$

B $m_e = \frac{9.8 \times 6.7 \times 10^{-11}}{4.1 \times 10^{13}}$

C $m_e = \frac{9.8 \times 4.1 \times 10^{13}}{6.7 \times 10^{-11}}$

CORRECT ANSWER: The mass of the Earth is:

$$m_e = 6.0 \times 10^{24} \text{ kg}$$

As a check, we looked up the accepted value for the mass of the Earth in the Handbook of Chemistry and Physics, 44th edition (1963), p. 3486. The accepted value is:

$$m_e = 5.983 \times 10^{24} \text{ kg}$$

Pretty good agreement, we'd say! You couldn't ask for much better!

NOTEBOOK ENTRY
Lesson 9

(Item 1)

(f) By solving the equation above, $g = G\frac{m_e}{r^2}$, for m_e, we obtain an expression which allows us to calculate the mass of the Earth directly.

$$m_e = \frac{gr^2}{G}$$

in which $g = 9.80$ m/sec^2, $r = 6.44 \times 10^6$ meters, and $G = 6.67 \times 10^{-11}$ m^3/kg-sec^2.

Now for a problem about Mars, turn to page 115.

A man who weighs 220 lb on Earth has a mass of about 100 kg, since 1 kg of mass weighs about 2.2 lb. Again, just for the fun of it, suppose we find out how much this man would weigh on Mars.

Since <u>weight is the force of gravitation</u>, then <u>w</u> may be substituted for F_g in the gravitational equation. If m_m = mass of Mars, r_m = radius of Mars, and m_p = mass of the person, then which equation would you use to find his weight on Mars?

(22)

A $w = m_p g$

B $w = G\frac{m_p m_m}{r_m^2}$

C $w = G_m \frac{m_p m_m}{r_m^2}$

YOUR ANSWER --- B

You are correct. In collecting terms, we have:

$$w = \frac{6.7 \times 6.4}{(3.4)^2} \times \frac{10^{-11} \times 10^2 \times 10^{23}}{(10^6)^2}$$

$$= \frac{6.7 \times 6.4}{(3.4)^2} \times 10^2$$

$$= \frac{6.7 \times 6.4}{11.6} \times 10^2$$

$\underline{w}$ on Mars $= 3.7 \times 10^2 = \underline{370}$ $\underline{\text{newtons}}$

You will remember that a 100 kg man weighs about 220 lb on the Earth. So, about how many pounds does the same man weigh on Mars? (In the event that you have forgotten the number of newtons in one pound, we'll refresh your memory: the conversion factor is 4.5 nt/lb.)

Write your answer to two significant figures; then turn to page 84.

YOUR ANSWER --- C

You are correct. Since m_e, G, and m_r appear unchanged in both equations, then you may consider them constant, of course. So to make our work a bit simpler, let us now rewrite the two equations using k in place of Gm_em_r.

At Sea Level:	At the New Altitude:
$w = \frac{k}{(4,000)^2}$	$\frac{w}{2} = \frac{k}{r^2}$

See how nicely this kind of substitution simplifies the aspects of the equations?

So we have two equations. Only one of these contains the unknown, the quantity in which we are interested, namely r. (Remember, we are interested in the altitude to which the rocket must ascend in order to halve its weight.) A trick that is often used in eliminating undesirable unknowns in a pair of equations is the process of dividing one equation by the other, thus enabling us to cancel similar quantities.

We want you to do this. Divide the "sea level" equation by the "new altitude" equation. Which one of the following can be obtained this way?

(26)

A $2 = \frac{r^2}{(4,000)^2}$

B $\frac{1}{2} = \frac{r^2}{(4,000)^2}$

YOUR ANSWER --- D

You are correct. Taking the equation given and transposing we have:

$$(4{,}000)^2 \times 2 = r^2$$

Then, taking the square root of both sides, we obtain:

$$r = 1.41 \times 4{,}000 \text{ miles}$$

The product is r = 5,640 miles.

So, the new distance between the rocket and the center of the Earth required to halve the original sea level weight of the rocket is 5,640 miles.

Using this figure and the known radius of the Earth, the altitude required to halve the weight of the rocket is 1,640 miles. This is obtained merely by subtracting the radius of the Earth (4,000 mi) from the "new distance" (5,640 mi), or 5,640 mi - 4,000 mi = 1,640 mi.

Please turn to page 25.

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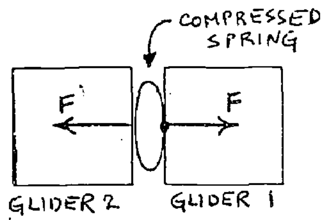
This page has been inserted to maintain continuity of text. It is not intended to convey lesson information.

WORKSHEET

Please listen to Tape Segment 1 for this lesson. You will also need a single-concept film projector and the film entitled

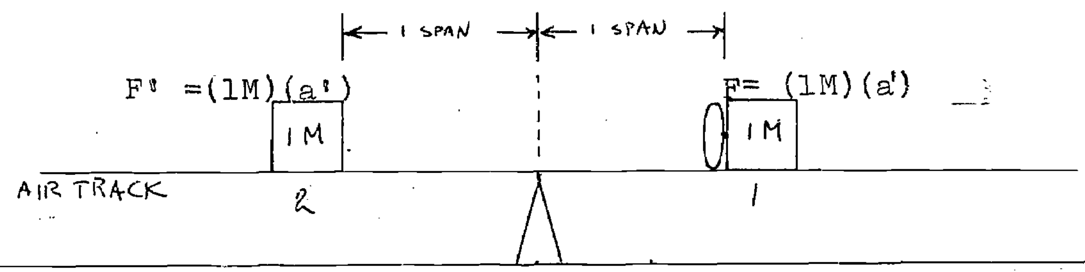
NEWTON'S THIRD LAW

DATA ITEM A: Diagrams below.

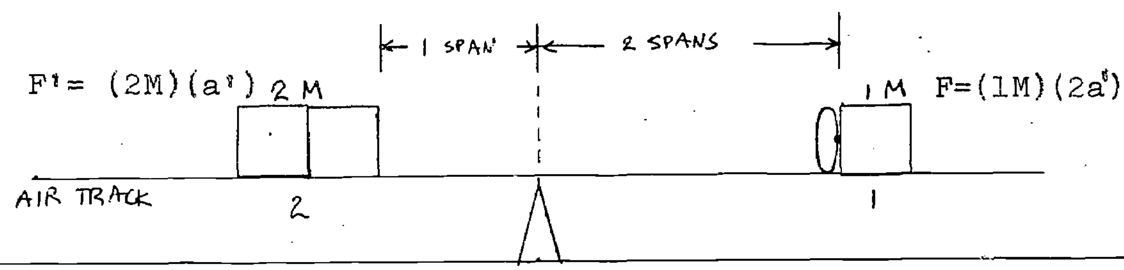


The flat spring is attached to Glider 1 on the right. When compressed, it causes Glider 1 to exert force F' on Glider 2, and Glider 2 to exert force F on Glider 1. Either force may be considered to be the reaction to the other.

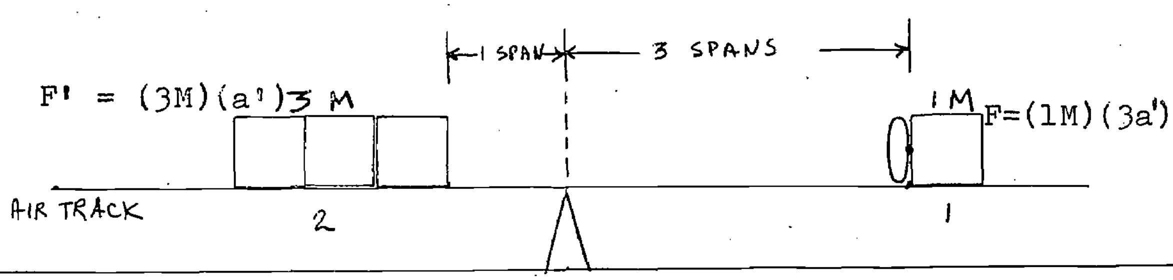
TRIAL 1



TRIAL 2



TRIAL 3



(Go to next page, please)

Let's look at the diagram for Trial 1 on the previous page. We start with two equal masses, i.e. each glider has a mass of 1M. When the air is turned on, the forces produced by the compressed spring cause the gliders to move apart. Each one moves a distance of 1 span. Now, you know that the distance an object will be displaced is related to the acceleration and time by the equation:

$$s = \frac{1}{2} at^2$$

Solving this for acceleration, we have:

$$a = \frac{2}{t^2} s$$

The interval between the time that the air is turned on and the gliders begin to move and the time that the air is turned off to stop the gliders is clearly the same for both gliders. Therefore, time t is a constant for both gliders. Since the number 2 in the numerator is also a constant, then we can replace

$\frac{2}{t^2}$ by k and write

$$a = k\,s$$

Thus, the acceleration is directly proportional to the distance covered by the corresponding glider. If one glider travels twice as far as another, it must have had twice the acceleration; if one travels three times as far as the other, it must have had three times the acceleration; and so forth.

So, coming back to Trial 1 we have:

for the glider on the right (Glider 1): $F = (1M)(a')$

$F = Ma'$

For the glider on the left (Glider 2): $F' = (1M)(a')$

$F' = Ma'$

Therefore, $F = F'$ and we see that the action force, say F, is equal to the reaction force F', thus proving Newton's Third Law for this particular case.

In the second trial (Trial 2), the 1M glider moved twice as far as the 2M glider, hence the 1M glider must have had twice the acceleration of the 2M glider so we can write

for Glider 1: $F = (1M)(2a') = 2Ma'$

for Glider 2: $F' = (2M)(a') = 2Ma'$ so again, $F = F'$

(go on to the next page, please)

QUESTIONS

Use the special AV Computer Card to indicate your answer choices.

1. In Trial 3, the fact that Glider 1 moved 3 times as far as Glider 2 (3 spans as compared with 1 span) indicates that

   A glider 1 had three times the mass of glider 2.
   B glider 2 moved with three times the velocity of glider 1 throughout the trip.
   C more force was acting on glider 1 than on glider 2.
   D glider 1 had three times the acceleration of glider 2.
   E none of the above statements is correct.

2. For Trial 3, which one of the statements below is correct?

   A The action and reaction forces acting on Glider 1 are equal and opposite.
   B The action and reaction forces acting on Glider 2 are equal and opposite.
   C The action force on Glider 1 equals the reaction force on Glider 2 and is opposite in direction.
   D The action and reaction forces act in the same direction but on two different bodies.
   E None of the above statements is correct.

3. Suppose we ran a fourth trial. Being finally convinced that $F = F'$ for this apparatus, suppose we made Glider 2 a <u>5M</u> glider and Glider 1 a <u>2M</u> glider. Then if the acceleration of Glider 2 was again taken as a', what would be the acceleration of Glider 1?

   A $2.5\ a'$
   B $5\ a'$
   C $1\ a'$
   D $0.40\ a'$
   E $10\ a'$

Please return now to page 3 of the Study Guide for Lesson 9.

WORKSHEET

Please listen to Tape Segment 2 before starting on this Worksheet. Answer choices should be made on the Computer card.

4. When we say that the law of gravitation is universal in nature, we imply that

A every object in the universe exerts an equal force on every other object in the universe.
B gravitation and centripetal force are identical forces throughout the universe.
C all forces exerted on one body by another in any part of the universe are gravitational forces.
D beside the Sun, all other stars in the universe exert significant forces on the Earth, hence have a significant effect on its orbit.
E the orbit of a planet circling a star that is a billion billion billion miles away is predictable by the same law as the orbit of the Moon around the Earth.

5. Two bodies, A and B, are small fixed spheres having masses in the ratio of 4 to 1. At what point on the line joining the centers of A and B should a third small (C) sphere be placed so that the gravitational forces of A and B on C are in equilibrium?

A Twice as far from B as it is from A.
B Twice as far from A as it is from B.
C Four times as far from B as it is from A.
D Four times as far from A as it is from B.
E Sixteen times as far from A as it is from B.

6. The Earth attracts the Moon with a force: $F_g = k \frac{m_M m_E}{r^2}$.

The mass of the Moon is roughly 1/80 that of the Earth. Therefore,

A the Moon attracts the Earth with a force $\frac{1}{80} F_g$.
B the Moon attracts the Earth with a force exactly equal to F_g.
C the Earth provides the centrifugal force needed to keep the Moon in orbit.
D a man on the Moon would weigh 1/80 his Earth weight.

Please return now to page 96 of the STUDY GUIDE.

Please listen to Tape Segment 3 before starting on this Worksheet. You will also need a single-concept projector and the film entitled: MEASUREMENT OF G - THE CAVENDISH EXPERIMENT.

Data Item A: The value of G may be taken as $6.67 \times 10^{-11} \frac{m^3}{kg\text{-}sec^2}$.

QUESTIONS

7. Why is a light pointer used in the experiment you have just seen rather than a metal pointer attached directly to the center of the bar of the dumbell?

A It's easier to see a light than a metal pointer.
B The light does not add an inertial mass to the system as a metal pointer would.
C The reflected light is sharper than a pointer would be.
D The light pointer and mirror system magnifies the tiny motion of the dumbell.
E The light pointer permits the experiment to be carried on in a darkened room.

8. The unit for G may be given in another way than that shown in Data Item A above. Which one of the following is the correct alternative unit?

A nt-m/kg

B $nt\text{-}m^2/kg$

C $nt\text{-}m^2/kg^2$

D $nt^2/m^2\text{-}kg$

E $nt/m^2\text{-}kg^2$

Please return to page 57 of the STUDY GUIDE.

HOMEWORK PROBLEMS

Lesson 9

Physical constants that may be required for solving these problems:

g (at the surface of the Earth) = $9.8\ m/sec^2$

$G = 6.7 \times 10^{-11}\ m^3/kg\text{-}sec^2$

$\pi = 3.14$ and $\pi^2 = 9.9$

1. Prove that the time required for one revolution of a body moving in a circle of radius r is given by

$$T = \frac{2\pi r}{v}$$

where T = the time for one revolution (called the period) in sec, if r is the radius of the circle in meters and v is the uniform tangential velocity in meters per second.

2. Frequency is the reciprocal of period. Write the expression for the frequency of rotation in terms of the radius of rotation r and the uniform tangential velocity v. (Use f as the symbol for frequency).

3. What is the force of gravitational attraction between a body having a mass of 50 kg and a second mody with a mass of 60 kg when their centers are separated by 2 meters.

4. How much gravitational force does the Earth exert on a 50-kg mass at sea level?

5. What is the value of g at a point in space that is 7.0×10^6 m from the center of the Earth? (The mass of the Earth may be taken as 6.0×10^{24} kg).

6. What is the weight of a 1000-kg rocket when it is 7.0×10^6 m from the center of the Earth?

7. At a point 6.7×10^6 m from the center of the Earth, g may be taken as $9.0\ m/sec^2$. What velocity must be given to an Earth satellite to send it into a circular orbit at this distance?

8. The mass of the Earth is about 80 times that of the Moon. A rocket fired toward the Moon has traveled two-thirds of its journey. At that point, what is the ratio of the Earth's force of attraction on the rocket to the Moon's force of attraction on the rocket?

Please listen to Tape Segment 4 before starting this Worksheet.

QUESTIONS

9. Despite all we have said about the equality of free-fall acceleration of different masses at the same point near the Earth, it is still quite obvious that an acorn falls much faster than a leaf from the same branch of a tree. How do you account for this?

A The air retards the leaf more than the acorn.
B The leaf has more inertia due to its shape.
C The leaf has less mass than the acorn.
D Atmospheric pressure causes this effect.
E The leaf has very tiny holes in it.

10. An aluminum ball and a lead ball are dropped from the top of a building at the same time. The balls both have smooth surfaces and exactly the same diameter. Both balls are observed to strike the ground at the same instant. Which of the statements below relative to this experiment is a FALSE statement?

A The accelerating force acting on the lead ball is greater than the accelerating force acting on the aluminum ball.

B The aluminum ball has a smaller mass than the lead ball.

C The lead ball has more inertia than the aluminum ball.

D Both balls have the same average acceleration throughout the entire trip.

E The accelerating force acting on both balls is identical.

11. Which one of the following is an acceptable definition of mass?

A Velocity per unit force.
B Force per unit acceleration.
C Force per unit velocity.
D Acceleration per unit force.
E Displacement per unit force.

Please return now to page 19 in the STUDY GUIDE.